

1. What is the instantaneous rate of change at  $x = 3$  of the function  $f$  given by

$$f(x) = \frac{x^2 - 2x}{x+1}$$

$$f'(x) = \frac{(x+1)(2x-2) - (x^2-2x)(1)}{(x+1)^2}$$

$$f'(3) = \frac{(4)(4) - 3}{16}$$

$$f'(3) = \frac{13}{16}$$

2. If  $a \neq 0$  then  $\lim_{x \rightarrow a} \frac{x-a}{x^3-a^3} = \frac{a-a}{a^3-a^3} = \frac{0}{0}$  indeterminate, so fuss...

$$= \lim_{x \rightarrow a} \frac{x-a}{(x-a)(x^2+ax+a^2)}$$

$$= \lim_{x \rightarrow a} \frac{1}{x^2+ax+a^2} = \frac{1}{a^2+a^2+a^2} = \frac{1}{3a^2}$$

3. When is the graph of  $y = x^4 - 3x^2 + 5x - 2$  concave down?

$$y' = 4x^3 - 6x + 5$$

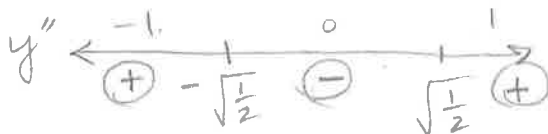
$$y'' = 12x^2 - 6$$

$$0 = 6(2x^2 - 1)$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{1}{2}}$$

$$x \approx \pm 0.7$$



Concave down on  $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

4. Determine the value of  $m$  that makes the following function continuous

$$f(x) = \begin{cases} mx^2 + 2 & \text{for } 0 \leq x \leq 2 \\ x + 8 & \text{for } 2 < x \leq 4 \end{cases}$$

Must be connected at  $x = 2$

$$m(2)^2 + 2 = 2 + 8$$

$$4m = 8$$

$$m = 2$$

5. Given  $y^2 = 2 + xy$ , find all points  $(x, y)$  on the curve where the tangent line has a slope of  $\frac{1}{2}$ .

$$2y \frac{dy}{dx} = 0 + x \frac{dy}{dx} + (1)y$$

$$\frac{dy}{dx}(2y - x) = y$$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

$$\frac{1}{2} = \frac{y}{2y - x}$$

$$2y - x = 2y$$

$$-x = 0$$

$$x = 0$$

$$y^2 = 2 + (0)y$$

$$y = \pm \sqrt{2}$$

$$\begin{matrix} (0, \sqrt{2}) \\ (0, -\sqrt{2}) \end{matrix}$$

6. Find  $y'$  given  $y = e^{2x^2+1}$

$$y' = 4x e^{2x^2+1}$$

7. Find the derivative of  $f(x) = \ln \frac{\sqrt{2x-1}}{x(x+2)^3}$

$$f(x) = \frac{1}{2} \ln(2x-1) - \ln x - 3 \ln(x+2)$$

$$f'(x) = \frac{\frac{1}{2}}{2x-1} - \frac{1}{x} - \frac{3}{x+2}$$

→ OK to leave as 3 separate fractions, but extra credit on test for correctly putting them together:

$$f'(x) = \frac{x(x+2) - (2x-1)(x+2) - 3x(x+2)^2}{x(x+2)(2x-1)}$$

$$f'(x) = \frac{x^2+x - (2x^2+4x-2) - 6x^2-12x-6}{x(x+2)(2x-1)}$$

$$f'(x) = \frac{-7x^2-11x-4}{x(x+2)(2x-1)}$$

8. Find the derivative of  $y = (2x)^{2x}$

$$\ln y = [2x] [\ln(2x)] \quad \text{Now product rule!}$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = [2x \cdot \frac{2}{2x} + 2 \ln(2x)] y$$

$$\frac{dy}{dx} = (2x)^{2x} (2 + 2 \ln(2x))$$

9. Find the following limit:  $\lim_{x \rightarrow 0} x^{\ln(x+1)} = 0^0 = 0^0$  Indeterminate

$$\text{let } y = \lim_{x \rightarrow 0} [x^{\ln(x+1)}]$$

$$\ln y = \lim_{x \rightarrow 0} [\ln(x+1) \cdot \ln x]$$

$$\ln y = \lim_{x \rightarrow 0} \left[ \frac{\ln(x+1)}{\frac{1}{\ln x}} \right] = \frac{0}{-\infty} = \frac{0}{0} \text{ so L'Hop. R.}$$

$$\ln y = \lim_{x \rightarrow 0} \left[ \frac{\frac{1}{x}}{\frac{1}{(\frac{1}{x})}} \right] = \lim_{x \rightarrow 0} \left[ \frac{1}{x^2} \right] = \frac{1}{0} = \infty$$

10. Find the following limit:

$$\lim_{x \rightarrow -1} \left( \frac{x}{x^2-1} - \frac{1}{x+1} \right) = \frac{-1}{0} - \frac{1}{0}$$

$$= -\infty - \infty$$

$$= \boxed{-\infty}$$

\*  $\infty - \infty$  is indeterminate, but  $-\infty - \infty = -\infty$